

Searching for features of a string-inspired inflationary model with cosmological observations

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The latest *Planck* results show a power deficit in the temperature anisotropies near $\ell \approx 20$ in the cosmic microwave background (CMB). This observation can hardly be explained within the standard inflationary Λ -cold-dark-matter (Λ CDM) scenario. In this paper we consider a string theory inspired inflationary model (axion monodromy inflation) with a step-like modulation in the potential which gives rise to observable signatures in the primordial perturbations. One interesting phenomenon is that the primordial scalar modes experience a sudden suppression at a critical scale when the modulation occurs. By fitting to the CMB data, we find that the model can nicely explain the $\ell \approx 20$ power deficit anomaly as well as predict specific patterns in the temperature-polarization correlation and polarization autocorrelation spectra. Though the significance of the result is not sufficient to claim a detection, our analysis reveals that fundamental physics at extremely high energy scales, namely, some effects inspired by string theory, may be observationally testable in forthcoming cosmological experiments.

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I. INTRODUCTION

With accumulated high-precision measurements of the cosmic microwave background (CMB) radiation [1–3], it is believed that our universe has experienced a dramatic phase of expansion at very early times as described by the inflationary Λ CDM cosmology from which a power law spectrum of primordial perturbations is obtained [4]. The recently released *Planck* data, however, reported deviations from a power law connected to a deficit at multipoles $\ell \approx 20 - 40$ in the temperature power spectrum [5]. Although this anomaly does not have sufficient statistical significance due to the cosmic variance at these multipoles, it is of enough interest to ask whether such an experimental signature provides a hint to new physics beyond Λ CDM.

In order to understand cosmological data from fundamental physics, we consider an important class of large field inflationary cosmology realized by string theory, which is dubbed axion monodromy inflation (AMI) [6, 7]. In the corresponding stringy setup, a number of axion fields coupled to fluxes can realize a super-Planckian field variation with a soft shift symmetry breaking along the axion potential due to couplings or D-branes. This is how a class of monomial potentials has been achieved [8]. This model and its extensions have drawn a lot of attention in the literature (e.g., see [9, 10] and references therein). In

particular, it was observed in [10] that axion monodromies can be obtained in terms of D3- and D5-branes with torsional cycles in which there exists at least one scalar mode that is free from the dangerous moduli stabilization effects from the Kähler potential. This scalar field, when applied to the early universe, can drive a sufficiently long phase of inflation where the potential parameters are sensitive to flux couplings, namely, the potential experiences a modulation at a critical value of the inflaton field. As a result, it provides an interesting implementation of step inflation [11, 12] from the perspective of string theory.

In this paper we aim at examining observational signatures of this type of inflationary model in cosmological surveys. Specifically, we present an estimate of the power spectrum of primordial curvature perturbations and analytically find that it possesses a suppression feature at a critical length scale. Using the *Planck* 2013 data, we perform a numerical analysis and find that this feature can nicely interpret the anomaly at low multipoles as observed in the recent CMB observations which cannot be explained by the standard model, i.e. Λ CDM. Afterwards, we compute the E-mode polarization spectra, and interestingly, we find that our model also predicts nontrivial patterns on these spectra. We take the convention of the *Planck* team throughout the paper.

II. MODEL

We work with the string-inspired inflationary model of axion monodromy in the presence of torsional cycles and flux couplings [10]. In this framework, the 4-dimensional effective action is described by a canonical scalar field with a polynomial potential coming from either the Chern-

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Simons term or flux couplings. Due to a variation of the flux coupling, the inflaton's potential can be expressed as

$$V(\varphi) = \lambda M_{\text{pl}}^4 (\varphi/M_{\text{pl}})^n \mathcal{F}_V(\varphi), \quad (1)$$

with

$$\mathcal{F}_V(\varphi) = 1 + \xi^2 / [1 + e^{-2c_H(\varphi^2 - \varphi_c^2)/M_{\text{pl}}^2}], \quad (2)$$

where M_{pl} is the reduced Planck mass (see [10] for the string theory implementation with $n = 2$).

The coefficient of the potential takes the form $(1 + \xi^2)\lambda$ in the ultraviolet (UV) regime ($|\varphi| > \varphi_c$) but becomes λ in the infrared (IR) regime ($|\varphi| \leq \varphi_c$), where ξ is a dimensionless parameter. The other dimensionless parameter c_H is associated with the smooth transition between the UV and IR regimes of the potential near the critical scale characterized by φ_c . As was observed in [10], the inflaton field undergoes standard slow roll dynamics in both the UV and IR regimes, but near the potential modulation, the slow roll condition is briefly broken.

In this case it is convenient to introduce the generalized slow roll (GSR) parameters [13], which up to i -th order are given by $\epsilon \equiv -\frac{\dot{H}}{H^2}$, $\eta_i \equiv \frac{\varphi^{(i+1)}}{H^i \dot{\varphi}}$, where a dot denotes a cosmic time derivative. Then one can follow the formalism of the GSR approximation to derive an analytic expression of the power spectrum. Here we skip the detailed procedure (which will be presented in a follow-up paper) but directly write down the power spectrum of primordial curvature perturbation as

$$\mathcal{P}_{\mathcal{R}}(k) = \bar{\mathcal{P}}_{\mathcal{R}}(k) \mathcal{F}_V(\varphi(k)) \mathcal{F}_M(k), \quad (3)$$

where $\bar{\mathcal{P}}_{\mathcal{R}}$ is the featureless power spectrum from the standard model and \mathcal{F}_M represents the modifications introduced by the GSR approximation. Similar to the standard case, the power spectrum can be parametrized by $\bar{\mathcal{P}}_{\mathcal{R}} \equiv A_s (k/k_*)^{n_s-1}$, where A_s and n_s are the amplitude and the spectral index, respectively, and k_* is the pivot scale. For a specific power law potential, we have

$$\begin{aligned} \varphi(k) &\approx \varphi_* - n \frac{M_{\text{pl}}^2}{\varphi_*} \ln\left(\frac{k}{k_*}\right), \\ \varphi_*^2 &\approx \frac{n}{2} (4N_* + n) M_{\text{pl}}^2, \end{aligned} \quad (4)$$

and the power spectrum quantities A_s and n_s can be approximately written as [5, 14]

$$A_s \approx \frac{\lambda}{12\pi^2 n^2} \left| \frac{\varphi_*}{M_{\text{pl}}} \right|^{n+2}, \quad n_s - 1 \approx \frac{-2(n+2)}{4N_* + n}. \quad (5)$$

To apply these relations it is convenient to introduce k_c at $\varphi(k_c) = \varphi_c$ where the feature is located. The factor \mathcal{F}_M grasps the signatures brought by the potential modulation, and therefore, its value equals unity away from k_c but exhibits the GSR feature near k_c . For $\xi > 1$, these features have an oscillatory behavior near k_c , analogous to [15] where a quadratic model with a different modulation form was considered.

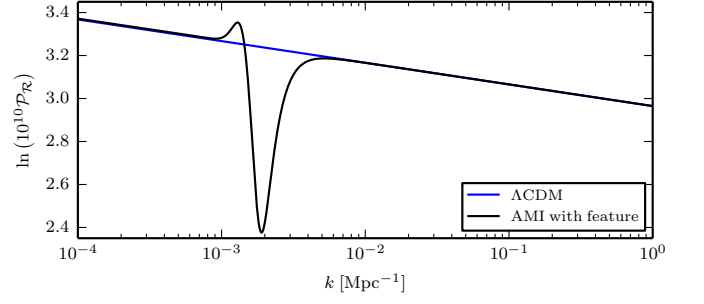


FIG. 1: Power spectra of primordial curvature perturbations $\mathcal{P}_{\mathcal{R}}$ as a function of the comoving wave number k . The blue curve shows the standard Λ CDM model and the black curve presents an example of the AMI with feature given by (3).

In this paper we focus our interest in the case of $\xi < 1$. Accordingly, the GSR feature is very smooth and is only located in the vicinity of the potential modulation and mainly depends on the evolution of ϵ . To leading order one has $\mathcal{F}_M \simeq 1 + (4f_\alpha - 2)\epsilon + 2f_\alpha\eta_1 + (3f_\alpha^2 + \frac{5\pi^2}{12} - 4)\eta_1^2 + (\frac{\pi^2}{12} - f_\alpha^2)\eta_2$, where $f_\alpha \equiv 2 - \ln 2 - \gamma$ with γ being the Euler-Mascheroni constant. This factor can be further parametrized as follows,

$$\mathcal{F}_M(k) \simeq 1 + \frac{9c_H^2 \xi^2 \varphi^2 [e^{\frac{2c_H}{M_{\text{pl}}^2}(\varphi^2 - \varphi_c^2)} - 1]}{M_{\text{pl}}^2 \cosh[\frac{2c_H}{M_{\text{pl}}^2}(\varphi^2 - \varphi_c^2)] [e^{\frac{2c_H}{M_{\text{pl}}^2}(\varphi^2 - \varphi_c^2)} + 1]^3}. \quad (6)$$

The parametrized form (3) together with Eqs. (2) and (6) gives the primordial power spectrum of the AMI with feature, which is described by three extra parameters on top of A_s and n_s : ξ , c_H , and k_c (or φ_c equivalently), as demonstrated in Fig. 1. However, one extra parameter remains undetermined, i.e. the e -folding number (N_*) at the pivot scale, since it involves detailed knowledge of the reheating theory following it [16, 17]. A different value of N_* leads to a different φ_c , altering the position of the feature. This shows that N_* is highly degenerate with, at least, k_c . In the literature, N_* is sometimes fixed for simplicity [5, 18], but this leads to preferred values of n , as seen from Eq. (5). For the sake of generality, we investigate both possibilities: having N_* fixed and having it as a free parameter. Additionally, the parameter space can be further reduced by imposing theoretical constraints, namely, the power spectrum must always be positive, i.e. $\mathcal{F}_M > 0$. We impose this constraint in all numerical calculations below.

III. METHODOLOGY

We wish to compare the theoretical predictions of our model and to test the validity of our parametrization of the power spectrum with the current data. We perform a global fitting by running the CosmoMC package [19], a

Markov Chain Monte Carlo (MCMC) parameter sampler. We modify the publicly available CAMB [20] Boltzmann code with our parametrization (3), which gives the power spectrum with a feature described by c_H , ξ , and k_c . Those are combined with the “vanilla” parameters of *Planck*’s one-parameter extension of the baseline Λ CDM model [5, 14] consisting of 7 parameters ($\Omega_b h^2$, $\Omega_c h^2$, τ , Θ_s , n_s , A_s , r) where $\Omega_b h^2$ and $\Omega_c h^2$ are the baryon and cold dark matter densities, τ is the optical depth to reionization, Θ_s is the ratio of the sound horizon at decoupling to the angular diameter distance to the last scattering surface (multiplied by 100), n_s is the spectral index, A_s is the primordial amplitude, and r is the tensor-to-scalar ratio. As usual, we assume adiabatic initial conditions and a flat background universe.

Due to the limited knowledge of reheating, there remains uncertainty in determining the exact e -folding number N_* when the pivot mode (k_*) crosses the horizon. In order to take this into account, we first take the *Planck* convention [14] by setting $50 < N_* < 60$ and vary N_* when we perform the global fitting. Then, in order to have a closer look at the effect of fixing N_* on the parameter estimation, we perform three extra runs. Specifically, we fix N_* at 50, 55, and 60, which correspond to $k_* \approx 1.0$, 0.05, and 0.002 Mpc^{-1} according to the modeling of entropy generation at the end of inflation given by Eq. (24) in [14]. In this case, when the details of the entropy generation process are fixed, the relation between N_* and k_* is known. Thus, fixing k_* fixes N_* and vice-versa. redHere we emphasize again, in the case of fixing N_* , we only investigate a specific formula for reheating, the one that is adopted by the Planck collaboration [14]. As will be demonstrated below, the sensitivity of the feature in different multipole ranges is expected to depend on the choice of k_* , so it is crucial to set the relationship between N_* and k_* consistently. This is important in its own right, but the exploration of different reheating processes is beyond the scope of this paper.

When considering N_* as a free parameter, we do not assume any specific reheating scenario. This means that the relation between N_* and k_* is not known. In this case, the pivot scale k_* is fixed at the value $k_* = 0.05 \text{ Mpc}^{-1}$, which is equivalent to “marginalizing” over the uncertainty of the reheating mechanism. However, it is important to know N_* since it changes the position of the possible feature, and this is determined by the MCMC analysis.

We use the *Planck* 2013 TT power spectrum, both the low- ℓ ($2 \leq \ell < 50$) and high- ℓ ($50 \leq \ell \leq 2500$) multipole data. In order to break the degeneracy between reionization optical depth and the primordial amplitude, we include the WMAP [21] low- ℓ polarization power spectrum ($2 \leq \ell \leq 32$). Besides that, we adopt the baryon acoustic oscillation data from BOSS “LOWZ” and CMASS-DR11 surveys [22] in order to obtain a better estimation on Ω_m and H_0 , etc. We name the above data compilation *Planck13* in the following analysis. Moreover, we implement a logarithmic prior for ξ and k_c , namely $\log_{10} \xi \in [-8, 0]$ and $\log_{10}(k_c/\text{Mpc}^{-1}) \in [-4, 0]$, to be

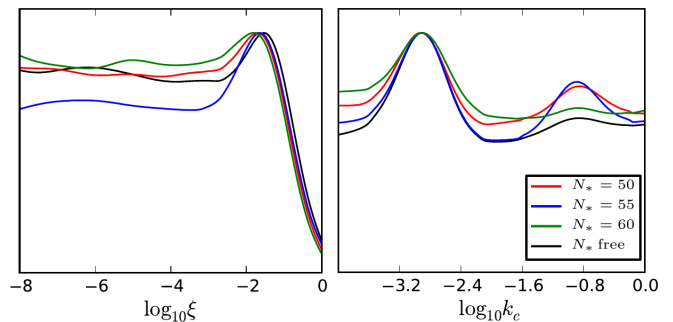


FIG. 2: One-dimensional posteriors of the feature parameters ξ and k_c for different e -folding numbers N_* as a result of the fitting to the *Planck13* data.

able to span a large parameter space. The theoretical constraint of $\mathcal{F}_M > 0$ restricts the value of c_H to be close to unity. Given this observation, we choose a flat prior for c_H such that $c_H \in [0, 1]$.

IV. RESULTS

After performing the global fitting of all parameters, we find that the best-fit values of the vanilla parameters are in agreement with the *Planck* values [3]. Then we present the posterior of ξ and k_c in Fig. 2. Since the current data cannot provide any significant constraint on c_H , we do not report any bound on it. We specifically consider the AMI with feature for the $N_* = 50$, 55, 60, and N_* free cases. The improvement in the maximum likelihood obtained in our runs with respect to Λ CDM is given by the quantity $\Delta\chi^2_{\text{eff}}$, which is found to be $-8.9, -11.6, -6.8, -6.2$, respectively.

We find that $k_c \approx 0.0013 \text{ Mpc}^{-1}$ is the best-fit value of the scale at which the feature in the primordial power spectrum is located. This roughly corresponds to the angular scale with multipole $\ell \approx 20$. Indeed, we see from Fig. 3 that for the $N_* = 60$ and N_* free cases, the suppression feature becomes manifest near $\ell \approx 20$, and accordingly, it leads to a slightly better fit to the *Planck* 2015 low- ℓ data [23]. Hence, our model can naturally explain the suppression anomaly near $\ell \approx 20$, and the interpretation is that this observed anomaly is due to a short phase of slow-roll violation during inflation [10].

We notice from Fig. 3 that in the high- ℓ regime our best-fit curves from *Planck13* data have a slight discrepancy with the *Planck* 2015 results. This is due to the fact that there is roughly a 2% shift in $A_s e^{-2\tau}$ between the 2013 [2] and 2015 [3] data induced by the absolute calibration parameter correction of the 143 GHz channel in the 2015 pipeline [24]. Moreover, our analysis reveals that the posterior of $\log_{10} k_c$ for the $N_* = 50$ and 55 cases can have a second peak away from the anomaly scale, hence they do not reproduce the $\ell \approx 20$ suppression in Fig. 3. The anticorrelation between N_* and k_* (the larger N_* is,

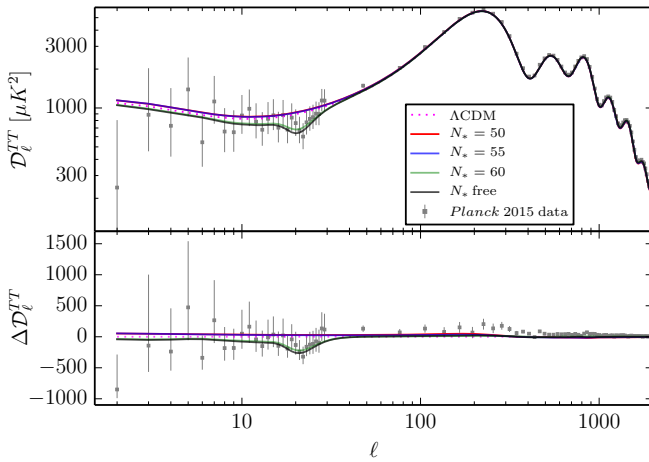


FIG. 3: The CMB temperature-temperature power spectra for the best-fit Λ CDM and AMI with feature models. Additionally, we superimpose the *Planck* 2015 data. The vertical axis of the top panel is defined as $\mathcal{D}_\ell^{TT} \equiv \ell(\ell+1)\mathcal{C}_\ell^{TT}/2\pi$. For the bottom panel, we plot the power spectrum residual ($\Delta\mathcal{D}_\ell^{TT}$) with respect to the Λ CDM one.

the farther away from the end of inflation Hubble crossing occurs, and hence the smaller k_*) demonstrates that in the case of small N_* (or large k_*) one can fit the small scale structures in the primordial power spectrum better and vice versa.

The current CMB data can only report an upper bound for the ξ parameter, $\log_{10} \xi \lesssim -1.2$ at 95% C.L., and it is insensitive to the c_H parameter compared to its theoretical prior. As for the degeneracy between feature and Λ CDM parameters, no significant correlation is obtained. The most interesting correlation is found between k_c and the tensor-to-scalar ratio ($r_{0.05}$). In the localized feature regime ($k_c \approx 0.0013 \text{ Mpc}^{-1}$), the tensor-to-scalar ratio is dragged to a greater value $r_{0.05} \lesssim 0.2$ at 95% C.L. This is because the potential modulation leads to a power deficit in the scalar spectrum without affecting as much the tensor spectrum (see [10]).

Several previous studies of CMB features reported $-\Delta\chi_{\text{eff}}^2 \sim \mathcal{O}(10)$. However, these signals were not statistically conclusive [5, 14, 18, 25–33]. The improvement obtained here is also not statistically significant, but yet, it shows that the localized feature is mildly favored by the data. As pointed out in [14], it is difficult to prove that these results are not coming from statistics overfitting of noisy data. Moreover, one needs to be careful with the $\Delta\chi_{\text{eff}}^2$ values found since the best-fit value provided by CosmoMC in its Metropolis-Hastings mode might not be fully trustworthy.

Observational signatures of the new model can also be analyzed within the CMB polarization data, namely, the E-mode spectra. On the one hand, since oscillations in the temperature maps are strongly washed out on small scales [34], the inclusion of the polarization maps will help us capture information of early universe models

at relevant scales. On the other hand, if the origin of the low- ℓ anomaly in the TT spectrum comes from the inflationary phase, a similar signal should also appear in the low- ℓ regime of the TE and EE spectra.

The *Planck* collaboration happened to release the polarization data very recently [23]. In Fig. 4, we show the TE and EE power spectra for the best-fit Λ CDM and AMI model with feature (obtained from the *Planck*13 data) and we make a first comparison with the *Planck* 2015 data. Looking at the residues of the TE and EE power spectra, one can see that our model is distinct from Λ CDM both at low- ℓ and high- ℓ . At low- ℓ , our model predicts power deficits around $\ell \approx 15-30$ in the TE and EE spectra (similar to the TT spectrum), of which the amplitudes ($\Delta\mathcal{D}_\ell^{TE}$ and $\Delta\mathcal{C}_\ell^{EE}$) are of order $\mathcal{O}(1)[\mu\text{K}^2]$ and $\mathcal{O}(5)[10^{-5}\mu\text{K}^2]$, respectively; while at high- ℓ the relative powers oscillate around Λ CDM in both the TE and EE spectra. The oscillations at $\ell > 30$ are due to small differences in the best-fit Λ CDM parameters via the mild correlations with the feature parameters, but the $\ell \approx 15-30$ suppressions in the TE and EE spectra are explicit predictions of the model compared to Λ CDM. Although these suppression signals in the E-mode spectra are difficult to be tested by the present observations since the differences are currently much smaller than the overall scattering in the data, they may provide a promising window for future CMB surveys which are expected to greatly improve their accuracy of polarization measurements.

V. CONCLUSIONS

Evidence of fundamental theory (e.g. string theory) occurs at extremely high energy scales, and hence, is difficult to be found directly by experiments. To search for evidence, it is important to investigate the associated cosmological implications, in particular, applications to the very early universe and their observational consequences. In this sense, various theoretical models have been proposed to explain certain CMB anomalies such as step potentials [18, 25, 26, 35–37], transient sound speed reduction [27–29, 38], massive fields [32, 33, 39], varying Planck mass models [40], pre-inflationary fast roll models [41], linear oscillations [30, 31, 42, 43], logarithmic oscillations [44–46], and cutoff models [47–49].

In the present paper, we studied the primordial power spectra of one class of inflationary model inspired by string theory and examined their patterns in the CMB. We interestingly found a suppression feature in the power spectrum of curvature perturbations in the vicinity of a critical scale, which could explain the power deficit anomaly of temperature anisotropies as indicated by the recent *Planck* data.

We further studied the angular power spectra of CMB polarization modes and performed a preliminary comparison with the *Planck* 2015 data. We showed that some patterns in these spectra are manifestly different from Λ CDM and hence the model is observationally distin-

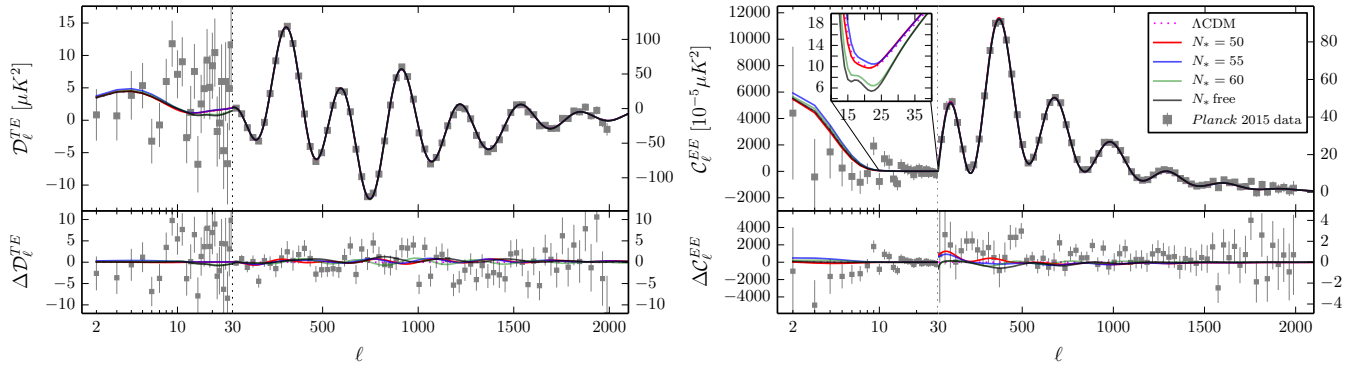


FIG. 4: Comparison between the theoretical CMB polarization power spectra (TE and EE) and the *Planck* 2015 data. In the left panel, we show \mathcal{D}_ℓ^{TE} and the residuals $\Delta\mathcal{D}_\ell^{TE}$ with respect to the Λ CDM best-fit. In the right panel, we show \mathcal{C}_ℓ^{EE} and $\Delta\mathcal{C}_\ell^{EE}$. Note that the scales differ for $\ell < 30$ and $\ell > 30$ as shown by the different vertical axis scales (the same way as Fig. 48 of [23]).

guishable if experimental accuracy is largely improved in the future. While these polarization signals could be important, it is also necessary to explore observational evidences from other avenues. For instance, we expect that the modulation in the inflaton's potential can give rise to specific non-Gaussianity signals (e.g., see [50–53]) as well as signals in the matter power spectrum that may be sensitive to the large scale structure experiments. The study of these topics will be important along with this present work.

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